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Experimental investigation of combined sway and non-sway buckling of frames

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ABSTRACT: A recently completed experimental investigation is described and shows the importance on collapse of rigid jointed frames when sway and non-sway critical modes exhibit simultaneous or nearly simultaneous critical loads. Results are directed towards the validation of a new theoretical model, which has the potential of improving design when more than one bucking mode and their associated imperfections can influence collapse.

1 BACKGROUND

1.1 *Simultaneous Buckling and optimisation*

Conventional design practice for redundant, rigid-jointed frames encourages designers to detail components to fail simultaneously in possibly a variety of different modes. Where failure modes involve buckling this practice can result in simultaneous buckling, in which the design loads for each component will be proportioned to have a consistent factor of safety to buckling collapse. One could envisage a "well designed" structural frame, at the factored design load, having component failures being triggered simultaneously over the entire frame. Even in a single column exhibiting a number of potential buckling failure modes, it is often a conscious goal of the designer to "optimise" material by striving to achieve simultaneity of buckling loads. Designing to achieve coincidence of buckling loads for minor and major axis buckling of a column, for example, can often be achieved by suitably changing the cross-section properties and lateral restraints so that the appropriate slendernesses are nearly equal. In a similar way, the overall bracing stiffness of a frame could be chosen so that the critical buckling loads in the sway modes occur at load levels close to those that control the critical loads of individual column buckling modes. It is the buckling phenomena that arise when more than one buckling mode occurs at similar load levels, that form the subject of the present paper.

1.2 *Simultaneous Buckling - Strong Interactions*

It is well known that in design against buckling the adoption of simultaneous or nearly simultaneous buckling loads can lead to extreme forms of imperfection sensitivity (see for example reference (1) in which the dangers of some forms of optimisation are highlighted). In these circumstances systems that in a single mode of buckling behave benignly can for the case simultaneous buckling take on characteristics more commonly associated with shell buckling. These strong interactions can occur even when the structure develops no material failure. In the present context a column having a very slender cross-section could develop simultaneously a local buckling of the cross-section and an overall buckling of the column. If this were to occur even the elastic buckling of the column could be transformed into one in which buckling takes on an extremely brittle form of failure at loads that are uncertain and require precise information on the shape and magnitude of any small initial imperfections.

It is for this reason that most design codes seek to avoid such interactions by insisting on, either, the cross-section shape being chosen to be sufficiently compact that local critical loads are well above overall critical loads (this is the practice for battened or latticed column, see for example ref (3) and (4)), or, the overall slenderness is sufficiently low that a local buckling occurs well before the onset of overall buckling (as is common in the design of sub-sea pressure hulls (5)). In either case the critical loads are forced to be sufficiently separate that a potentially deleterious, strong elastic interactive buckling involving both modes will not occur. However, these forms of strong interactive buckling are not the concern of the present work. The following is directed towards the rather weaker forms of interaction that can occur when imperfection sensitivity of buckling is driven by the development of material failure.

1.3 *Simultaneous Buckling - Weak Interactions*

Although maximum elastic buckling loads for columns are not sensitive to the levels of initial imperfection, the loads at which the non-linear elastic buckling response develops material failure can be highly imperfection sensitive. This is familiar in the conventional column buckling design curves. The knock-down factors observed in tests and predicted by theory are at their most extreme when the column has been designed to have simultaneity between the elastic critical buckling load and the so-called plastic squash load. It might be anticipated that when more than one elastic buckling is developing, as is the case when say sway and non-sway buckling modes of a frame occur at closely related elastic critical loads, an even stronger form of interactive elastic-plastic buckling behaviour could occur. It is this form of interactive elastic-plastic buckling that forms the focus of the following experimental studies.

2 APPROACHES TO COLUMN BUCKLING DESIGN

2.1 *Current Design Practice*

The majority of commonly adopted column buckling procedures (see for example references (4) and (5)) are either directly or indirectly based upon the classic work of Ayrton and Perry[6] who produced the first successful analysis of the interaction between the non-linear elastic response of an imperfect column and the development of first material failure. When the column is not subject to simultaneous axial load and lateral bending, the column buckling curves have been chosen to bound from below the scatter of many thousands of column buckling test results. While these design curves are closely informed by the Ayrton-Perry analysis and depend upon specified tolerances on column initial out-of-straightness, there is generally little explicit reference to the effects of imperfections and it would only be a determined designer who would be able to adjust the column buckling curves to allow for different levels of geometric imperfections or the imperfection effects that would arise from local bending when the column also carries primary moments. Even for single mode buckling the procedures become difficult to follow and consequently can easily be misinterpreted. Interactions between more than one potential buckling mode are virtually not considered, and yet, most design processes aim to achieve such simultaneity of buckling loads. With this interactive buckling being parametrically much more complicated than single mode buckling it is inconceivable that present design practice could easily be extended for it to be adequately covered. For this reason an alternative theoretical model has been developed[7][8]. This theoretical model is capable of covering a number of important classes of interactive elastic-plastic but in the present context is applied to frames for which a non-sway column buckling occurs at a load level similar to that required to produce sway buckling.

2.2 *A New Theoretical Model*

Since the new theoretical model of interactive sway and non-sway buckling has been fully described elsewhere[7,8], only those features essential to the present work will be described. The model is a direct extension to that of Ayrton - Perry[6] but assumes that there are now two buckling modes within which imperfection dependent non-linear elastic deformations are being developed. To calculate estimates of the actual maximum load carrying capacity of the frame, two closely related methods have been derived: the first predicts the load at which material plasticity commences; the second, predicts an estimate of the load corresponding to full-section plasticity. Both methods adopt the same essential steps, which are as follows:

- A convention linear analysis of the frame is undertaken to determine the moment distributions m^l. At every location these linear moments are used to define the 'squash load', P_p, which represents the axial load required to produce either first surface material failure or first full section plasticity (depending upon which criterion is being used for design);
- An idealised frame is used to calculate the critical loads (P_{cs}, P_{cn}) and associated modes for the lowest sway and non-sway buckling. These critical load analyses can usually be undertaken on the basis of a suitably simplified subframe;
- Define the amplitudes of the modal imperfections

ξ_s, ξ_n, from a straight forward interpretation of the linear bending response of step one above, together with appropriately chosen imposed modal tolerance limits and calculate the two imperfection parameters ($\rho_s \equiv \alpha_s \xi_s$, $\rho_n \equiv \alpha_n \xi_n$);

- On the basis of the two ratios (P_{cs}/P_p, P_{cn}/P_p) and the two imperfection parameters (ρ_s, ρ_n) read-off charts or calculate from eqn (19) or (26) of reference (8) the safe section buckling strength P_b, and ensure that it exceeds the factored designed load, P_d.

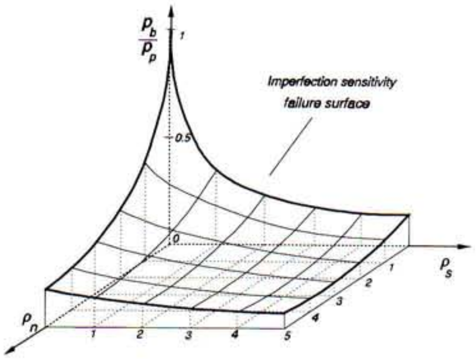


Figure 1. Effects of sway and non-sway imperfection parameters (ρ_s, ρ_n) on load for first yield, P_b, for P_{cn}= P_{cs}= P_p

For the case of $P_{cs} = P_{cn} = P_p$ the nature of the imperfection sensitivity surface is as shown in Figure 1. It is evident that both the sway and non-sway imperfection parameters have a strong influence upon the buckling loads, and that formoderate levels of imperfection parameters ($\rho \leq 2.0$) the buckling load can be as low as 0.2 times the squash load. Although Figure 1 shows the loads for first yield an almost identical surface results where the load for first full section plasticity, P_{fp}, is expressed in terms of a plastic squash load P_p^* and imperfection parameters (ρ_s^*, ρ_n^*) that are defined in slightly different but equally simple ways.

The present experimental programme was conceived to test the validity of these theoretical models.

3 EXPERIMENTAL PROGRAMME

3.1 *The Test - Rig*

To allow testing of as wide a parametric range as possible the intention was to test to destruction a large number of model frames. This meant that the frames should be small scale and constructed to allow simple and inexpensive replacement of any failed members. There should be easy control and monitoring of both the axial load and the secondary local loads used to vary the forms and magnitudes of the imperfections. Moreover, the method of axial load application should enable the axial load to re-

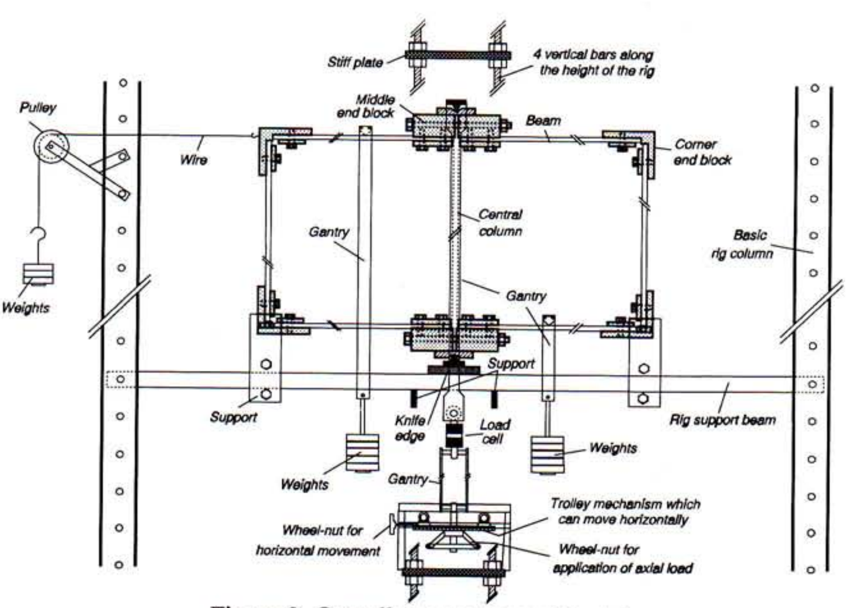


Figure 2. Overall arrangement of test-rig

main vertical and invariant during unimpeded sway deformations, and be applied through displacement control to allow monitoring of the otherwise unstable plastic unloading. The resulting rig is shown in Figure 2.

Displacement controlled axial load in the central column could be applied with adjustable eccentricity to allow variations of proportional loading imperfections[8]. By mounting the lower end of the axial load application gantry on a trolley mechanism the top end of the frame was able to undergo small sway deflections with a minimum amount of unintended resistance.

A fixed horizontal force applied at the top knee of the frame was used to introduce a non-proportional loading imperfection[8] in the sway mode, and further loading gantries on any one of the four beams allowed a controlled level of local beam loads to produce any desired level of non-proportional loading imperfections[8] in the non-sway column mode. The response was monitored through load cells and dial gauges mounted at various locations over the height of the central buckling column.

3.2 *Model Frame Design*

Model frames were constructed from standard hot rolled mild steel rectangular bars, connected together using specially designed friction grip corner and column end blocks. End supports to the column were designed to provide rigidity of the connection, but unimpeded column deformation over its full height. By avoiding welded connections not only were the costs of replacement frames kept low, but, more importantly, the vital plastic characteristics at the ends of the column were not affected by what would otherwise have been uncertain characteristics within the weld heat affected zone. A disadvantage of these removable rigid joints, was the complications they introduced into the theoretical modelling of the frame. However, it was easier to allow for these theoretical complications than it would have been to estimate the uncertain effects of welded joints in the small scale model frames. A major advantage of the friction grip joints was the ease with which failed central columns could be removed and replaced, and member replacements made to affect some desired changes in stiffness.

In order that the most severe range of imperfection sensitivities be tested it was intended to test frames having critical loads in the range $0.8<P_{cn}/P_{cs}<1.2$ and for which $P_{cn} \approx P_p$. By varying the beam lengths L_2 relative to those of the columns, $L_1 = L_3$, and by selecting appropriate second moments of area of the beams, I_2, and outer columns, I_3, relative to that of the central column, I_1, it was possible to cover a wide range of elastic critical load ratios. A typical parametric study is shown in Figure 3. Theoretical critical loads of Figure 3 make full allowance for the effects of the finite length rigid corner joints. By choosing three

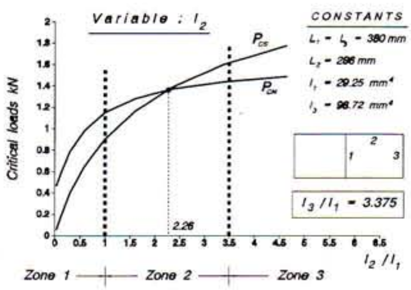


Figure 3. Typical parametric study used to design test models

Table 1 Geometric and critical buckling load properties

Frame class	Model	$L_1=L_3$ mm	L_2 mm	b_1 mm	b_2 mm	b_3 mm	d mm	P_{cS} kN	P_{cN} kN	P_y kN	P_{cS}/P_{cN}
	1	280	300	6	5	5	13	8.04	12.19	26.44	0.659
$\frac{P_{cS}}{P_{cN}} \leq 1$	2	270	350	5	5	5	13	6.53	8.66	22.45	0.754
	3	330	300	5	6	5	13	6.24	7.14	22.45	0.875
$\frac{P_{cS}}{P_{cN}} \approx 1$	4	340	314	5	6	6	13	6.75	6.75	22.45	1.000
	5	260	300	5	6	6	13	11.13	11.13	22.45	1.000
	6	210	250	3	5	3	13	5.20	4.46	13.85	1.170
$\frac{P_{cS}}{P_{cN}} \geq 1$	7	235	400	3	5	5	13	4.63	3.41	13.85	1.357
	8	235	370	3	5	5	13	4.89	3.45	13.85	1.416

different beam stiffnesses it was possible to define three classes of interactive behaviour. In zone 1 the sway buckling mode has critical load considerably less than the lowest critical load for non-sway modes; in zone 3 the situation is reversed. Over zone 2, the region of present interest, there is a close relationship between the lowest sway and non-sway critical loads. The three classes of model frame, whose geometries and material properties are summarised in Table 1, have been chosen to cover the behaviour exhibited in zone 2. The actual choices reflected the constraints of commercially available flat bar sizes. For each of these three classes of frame geometry a wide range of different combinations, of sway and non-sway, proportionate and non-proportionate loading imperfections, as well as a variety of sway and non-sway geometric imperfections could be tested.

3.3 *Test Procedure*

In setting-up the frames for testing, considerable care had to be taken to ensure adequate performance of the friction grip joint blocks and that the frame was aligned in the vertical plane. It was found to be essential to provide the adjustable trolley mechanism to allow the axial load to be maintained in a vertical alignment; in this respect a number of preliminary test results had to be abandoned as a result of uncertainty of the variable horizontal loads applied before the trolley mechanism was employed.

Once the frame was assembled the appropriate levels of horizontal load and local beam loads were applied to produce the desired levels of sway and non-sway imperfections. At least three separate tests were then carried-out on each frame. These allowed assessment of the effects of increasing levels of initial geometric imperfections resulting from the plastic residual distortions from the previous test. For each test sufficient loading and unloading cycles in the elastic range were undertaken to allow accurate experimental interpretation of both total equivalent imperfections and critical loads of sway and non-sway modes using an extended Southwell plot procedure. Once these experimentally derived imperfections and elastic critical loads were obtained, loading was continued to determine the elastic-plastic failure loads.

4 TEST RESULTS

4.1 *Elastic Tests*

On the basis of lateral deflections recorded over the length of the central column, it was possible to isolate the incremental deformation components in each of the sway and non-sway critical modes. Typical Southwell Plots for those separate modes are shown in Figure 4. Figure 4(a) shows the deformations associated with the sway modes for model 6, and Figure 4(b) those associated with the non-sway mode.

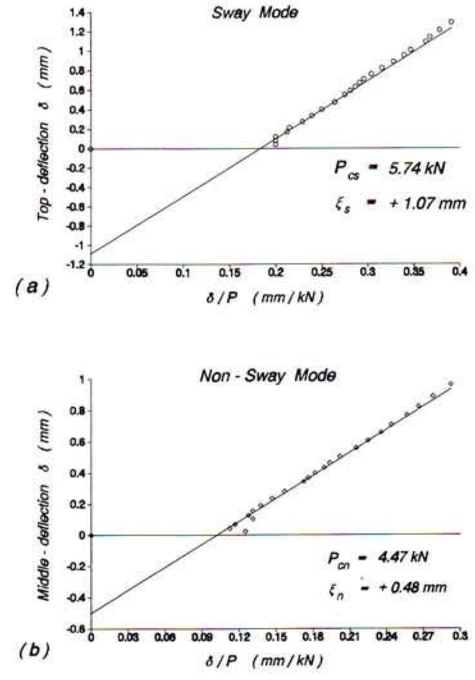
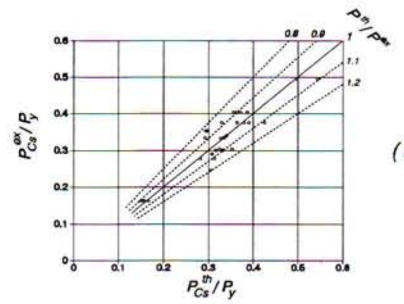
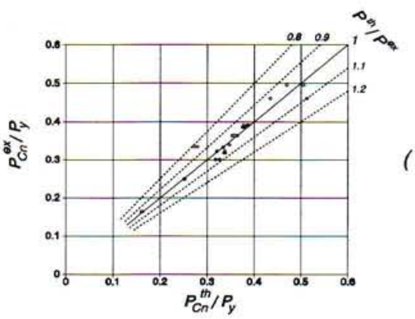


Figure 4. Interpretation of elastic test results using Southwell Plot to find (ξ_s, ξ_n) and (P_{cs}, P_{cn}) for test model 6

Despite the care taken to model the effects of the friction grip rigid connections there remained some uncertainties. Theoretical critical loads consequently differed from those obtained from the experimental interpretations using the Southwell plot approach. For model 6, shown in Figure 4, the theoretical critical loads were respectively 5.20 kN and 4.46 kN for the sway and non-sway modes. Because they also involved the inevitable but uncertain effects of friction, the sway critical loads tended to be a little less reliable than the non-sway. This is illustrated by the summarised results of Figures 5. For the sway and non-sway modes these correlate the experimentally determined elastic critical loads against those predicted theoretically. It is evident that scatter of the sway critical loads is greater than that



(a)



(b)

Figure 5. Correlations of the experimentally obtained critical loads and theoretical predictions for (a) sway and (b) non-sway critical loads

exhibited by the non-sway. In most cases the scatter can be observed to be within the ±10% band. However, to reduce this source of uncertainty from the prediction of buckling collapse loads, the experimentally observed critical loads (P_{cs}, P_{cn}) and total imperfections (ξ_s, ξ_n) are subsequently used for the theoretical estimates of buckling failure.

4.2 *Elastic - Plastic Buckling Failure*

Only when reliable results in the elastic range allowed interpretations to find the total imperfections (ξ_s, ξ_n) and elastic critical loads (P_{cs}, P_{cn}), was the loading increased to produce failure P_b. Failures, in the form of a maximum load, occurred fairly soon after the formation of first plasticity. Because the loading was essentially displacement controlled, it was possible to observe equilibrium states beyond the maximum buckling load, P_b. Once a clear maximum load had been recorded and small plastic deformations had been induced, the frame was unloaded. With these additional permanent deformations the frame was retested to find the new total imperfections and critical loads and subsequently a new maximum buckling load. Normally this process was repeated three times for each test frame.

Table 2 summarises the experimental observations and theoretical interpretations for one set of elastic - plastic tests on the 8 models of table 1. It is clear that the load for first yield P_{fy} represents as expected a lower bound of the experimentally recorded buckling collapse load P_b. That this lower bound can be as much as 25% represents the fairly considerable load reserve between the formation of first material failure and the numbers of fully formed hinges for

Table 2 Comparisons between experimental observations and theoretical predictions for elastic-plastic buckling.

Model	Experimental Results					Theoretical Results				Comparisons	
	P_{cS} kN	P_{cN} kN	ξ_S mm	ξ_N mm	P_b kN	ρ_s	ρ_n	P_{fy} kN	P_{fp} kN	P_{fy}/P_b	P_{fp}/P_b
1	8.58	11.47	1.05	1.05	8.20	1.02	0.83	7.09	7.67	0.86	0.94
2	6.92	8.42	0.75	0.70	6.70	0.75	0.72	5.98	6.31	0.89	0.94
3	7.03	7.54	1.35	0.12	6.5	0.82	0.14	5.66	5.94	0.87	0.91
4	7.44	7.32	1.60	0.86	6.02	0.63	1.01	4.84	5.60	0.80	0.93
5	11.17	11.33	1.05	0.75	8.70	0.47	0.87	7.95	9.34	0.91	1.07
6	5.40	4.67	0.67	0.40	4.20	-0.09	0.80	3.44	3.86	0.82	0.92
7	4.20	3.76	0.80	0.05	3.80	-0.40	0.10	2.83	3.07	0.74	0.81
8	4.13	3.49	0.85	0.85	3.00	-0.58	1.68	2.70	3.00	0.90	1.00

total collapse. A somewhat better basis for predicting collapse is the load corresponding with first fully formed plastic hinge P_{fp}. While this can not be guaranteed to produce a lower bound to collapse, it is evident that for the majority of tests it does provide a close estimates of collapse. In general the load for first hinge gives a more reliable prediction of the maximum buckling load.

Figure 6 summarises the results for all 28 tests on the 8 model geometries of table 1. Both the

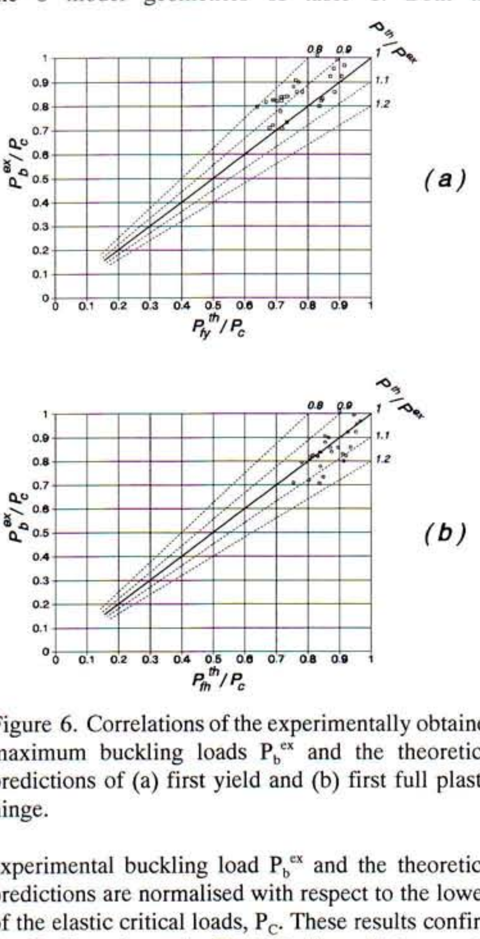


Figure 6. Correlations of the experimentally obtained maximum buckling loads P_b^{ex} and the theoretical predictions of (a) first yield and (b) first full plastic hinge.

experimental buckling load P_b^{ex} and the theoretical predictions are normalised with respect to the lowest of the elastic critical loads, P_C. These results confirm the findings shown in Table 2; first yield generally represents a reasonably reliable lower bound of collapse, with the scatter generally in the 0 to 20% scatter band. First full plasticity cannot be said to provide either an upper or lower bound wit scatter more evenly distributed over the ±10% scatter bands.

4.3 *Discussion of Results*

Although not directly shown in the experimental results reported, both the sway and non-sway modes contribute to the observed collapse behaviour. It was found that to provide sensible predictions of collapse the effects of both the sway and the non-sway imperfections were required. To base design on just one of these imperfections could seriously effect the reliability of theoretical estimates of buckling. While it is not within the scope of the present paper it is suggested that current design practice is insufficiently precise as to the incorporation of the effects of even a single modal imperfection. If the effects of both sway and non-sway imperfections are to be adequately incorporated into future design it is suggested that a theoretical model along the lines of that described in reference (8) will be required.

Plastic knock-down factors shown in Figure 6 are significant, but small compared with those that could be anticipated had elastic critical loads been closer to the squash load P_y. As indicated in Figure 5, the majority of the test frames had P_c/P_y in the range 0.3 to 0.5. This limitation was imposed by the capacity of the small scale test rig employed. Had geometries been tested in which the ratio P_c/P_y was closer to unity, the knock-downs would have been considerably greater.

5 CONCLUSIONS

Results from a test programme on rigid jointed frames, having sway and non-sway buckling modes occurring with similar elastic critical loads, demonstrate the importance of including the imperfection effects from both modes in theoretical estimates of collapse. Of the two theoretical models considered the lower boundedness of that based upon first material yield would commend it for future design. However, if future design is to adequately capture the effects of interacting buckling modes it will be necessary to reformulate it to allow more convenient and explicit incorporation of the influences of all modal imperfections.

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